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ABSTRACT

This study was designed to determine if it was feasible and appropriate to place the concept of integers as directed distances in the curricula for students in grades 3-5. Also investigated was whether the level of understanding of left-right conceptualization affected the understanding of the concept of integers and if sex classification affected the understanding of the concept of integers. A nine-day treatment was conducted by regular classroom teachers using an inductive teaching method. Prior to treatment, each subject was tested to determine the level of left-right conceptualization. The subjects were 49 third-grade students, 56 fourth-grade students and 52 fifth-grade students. Analysis revealed a significant difference in achievement among levels of left-right conceptualization in grades 3-5. The investigators concluded that teaching the concept of integers as a directed distance in the third grade would not be as advisable as in the fourth and fifth grades. (MK)

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A PROBLEM OF TEACHING THE CONCEPT OF INTEGERS AS A DIRECTED DISTANCE IN GRADES THREE THROUGH FIVE AND ITS RELATIONSHIP TO LEFT-RIGHT CONCEPTUALIZATION LEVEL, AGE LEVEL AND SEX

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Introduction

During the past four years, educators have begun to question the effectiveness of the so-called "Modern Mathematics" movement. It appears that this has been caused by the much publicized decline in test scores in mathematics, which along with the decline in reading scores, has brought about the "back to the basics" movement. This has been interpreted to mean "cut out the frills, teach more mathematics, to more children, in a shorter length of time." To one familiar with curriculum design of the past twenty years, all this brings to mind the Cambridge Conference held in the Summer of 1963.

The Cambridge Conference on School Mathematics (1963) proposed a curricula for mathematics which by the end of the century would have the effect of compressing the current curriculum to the extent that a student who had worked through thirteen years of mathematics in grades K-12, should have a level of training comparable to three years of college training, i.e., he/she would have the equivalent of two years of calculus, one semester of modern algebra, and one semester of probability theory.

In order to compress three years a new organization of the subject matter seems to be in order. The members of the Cambridge Conference outlined topics to be included in the mathematics curricula which would approach the goal they set. The section of their outline which is of interest to this study is the part pertaining to the introduction of the integers.

According to the Cambridge Conference Report, one of the major objectives of mathematics instruction in the elementary school should be an understanding

of the real number system. To accomplish this objective, the members of the Conference recommended that from the beginning of mathematical instruction the number line be used to represent the real numbers with the positive and negative integers shown and used on it. The Conference members felt that the operations of signed numbers should be taught in grades three through six.

Contemporary curricula do not generally include the integers until approximately the junior high school level. An examination of nine public school mathematics textbook series for grades 3-6 revealed one series which introduced integers in the third grade, two in the fifth grade, five in the sixth grade and one which did not include integers. The series that introduced integers in the third grade placed them near the end of the text as an enrichment topic through grade five. Both of the fifth grade texts and three of the sixth grade texts also placed the concept near the end of the text to be used as an extension of the textbook or as an enrichment topic for advanced students.

Before the integers are incorporated into the elementary curricula as suggested by the Cambridge Conference Report, there are many questions which need answers, among which are: 1) Is the study of integers feasible for the elementary school student? 2) At what stage in the child's development would it be best in terms of student understanding to introduct the concept? 3) At what level in the curricula should the concept be introduced? This study attempted to answer these questions with particular focus on students in the third through fifth grades which were selected because the age groups are approximately eight years through ten years, and according to Piaget (1928), the conservation of number is not achieved until age eight.

Statement of the Problem

The concept of integers is twofold. An integer can be conceptualized as

a directed distance or as an additive inverse. The concept of additive inverse is based upon two numbers whose sum is the additive identity zero, and for this an understanding of absolute value is required. Absolute value is based upon distance from zero, therefore an understanding of the concept of directed distance is a prerequisite for understanding additive inverse. Accordingly, this study was confined to the concept of an integer as a directed distance.

Since the directions left and right are used extensively when the concept of integers is presented as a directed distance, this study was also concerned with the possible effects of the level of left-right conceptualization on integer conceptualization.

Piaget (1928) tested 240 subjects, aged four through twelve, from a Swiss middle-class urban population on the concept of left and right. The study was divided into subtests which required the subject to identify: 1) parts of the subject's own body as being left or right, 2) parts of the examiner's body as being left or right of the examiner's body (the examiner was facing the subject), 3) relations of two objects, and 4) relations of three objects. The subject passed the subtest only if each question on it was answered correctly. Spontaneous correction and corrections made after errors of inattention were scored as correct.

Considering that a task is solved at a given age when at least 75 per cent of the subjects at that age gave the correct response, Piaget identified three stages in left-right conceptualization. In the first stage, the child relies on his own point of view in distinguishing left and right. In this stage, acquired at approximately five years of age, the ability to designate parts of one's own body in terms of left and right is acquired. In the second stage, the child is able to put himself in another's point of view and thus

recognizes left and right of a person facing him. According to Piaget, this ability is acquired at about age eight. In the third stage, the child is able to consider left and right from the point of view of objects themselves. He can distinguish between the relative term "to the left" and "to the right" with three objects. Piaget reported that this ability is acquired at about age eleven.

Laurendeau and Pinard (1970) working with 400 children in Montreal did a series of five tests dealing with the development of the concept of space. One of the tests was on left-right conceptualization. The subjects used in the study were distributed according to sex, father's occupation, level in school (or number of children in family in cases of preschool children), so as to reproduce exactly the proportions established by the French Canadian population census for the city of Montreal. The ages of the subjects ranged from two to twelve years inclusively, and were classified into six-month intervals up to the age of five years and at twelve-month intervals after that point. Each age interval contained fifty children.

The test used was divided into three sections, each designed to reach one of the three principal phases in the progressive structuring of the concept, which Piaget recognized. The test differs from those used by Piaget (1928) and Elkind (1961) mainly by the number of questions asked in each section and the order of questions in the sections. Piaget and Elkind used only four questions for the sections dealing with parts of the subject's own body and parts of another person's body. Laurendeau and Pinard used six questions for each section; thus, they claim, reducing the danger of success by chance. Laurendeau and Pinard alternated the questions not only with the left-right dimension but also with the parts of the body involved. Therefore, the subject

never identified the same part twice in a row. Again this appeared to reduce the effects of chance by allowing more independence of the child's response.

The results of the study differ very little from Piaget's and Elkind's. Table I gives the results in terms of median age of those successful in each section as well as the age of accession of the sections. It should be noted that Laurendeau and Pinard used a criterion of 50 per cent successes instead of the 75 per cent required by Piaget and Elkind for the age of accession.

Table I

Laurendeau and Pinard Study Results

	Own Body	Another Person	Three Objects
Median age	9:0	9:9	10:5
Age of accession[a]	5:1	7:10	9:9

[a]50 per cent successful

If 75 per cent are required to reply correctly to all questions to determine the age of accession, the age levels change to about seven years, ten years, and twelve years, respectively, for the three levels of left-right conceptualization. Laurendeau and Pinard explained the difference between these ages and those found by Piaget and Elkind by the smaller number of questions used by Piaget and Elkind.

This study also investigated the effects of sex classification on understanding the concept of integers as a directed distance. While there is no evidence of any difference in overall intelligence due to sex differences

(Fox, 1976; Fennema and Sherman, 1976), there is evidence of differences in intellectual areas (Stone and Church, 1957). At all ages, males usually perform better in the realm of quantitative and spatial relationships and females in the verbal sphere (Kagan, 1969; Maccoby and Jacklin, 1974).

In essence the basic problems of this study were to determine, first, if it were feasible in terms of student understanding to place the concept of integers as a directed distance in the elementary school curricula for students in grades three through five; second, if it was feasible, at which level was the concept of an integer as a directed distance appropriate; third, if the level of understanding of left-right conceptualization affected the understanding of the concept of integers; and fourth, if sex classification affected the understanding of the concept of integers.

Need for the Study

If the mathematics curricula is to be compressed as recommended by the Cambridge Conference Report, the structure of the real number system should be developed throughout elementary curricula with as little unnecessary repetition as possible. The present curricula contains a great deal of repetition by presenting the structure of the natural numbers, the rational numbers, and the integers, separately.

Adler (1972) stated that one of the principal defects in the curricula for arithmetic occurs when the concept of negative integers is not developed early enough. He suggests that within the next decade we try to eliminate this defect. He maintains that the concept should be introduced in the lower elementary grades and then be built upon until the full structure of the system of integers is revealed. Cochran (1966) feels that signed numbers will be creeping into the teaching of arithmetic more and more. However, Lovell

(1971) feels that the concept of negative integers might well be left until after the primary grades. He suggests that the teaching of negative integers in the third grade should be examined very carefully.

Piaget's work on child development and conceptualization has been interpreted to indicate that a child will be an adolescent before he can formally elaborate the concept of the set of integers (Lovell, 1971). The question remains, however, does the child need to be able to understand the complete ramification of the system of integers, especially negative integers, before the integers are introduced? Just as with the concept of zero, can we not introduce the concept of integers at different levels of curricula and with different degrees of sophistication of the concept? Lovell (1972) contends that we should not wait until pupils are ready to assimilate fully and formally. He maintains that the more familiar students are with the content, the more readily formal-operational thought can be elicited.

The emphasis being placed upon curriculum development, the concern for reorganization of mathematics curricula, especially with respect to the structure of the number system, along with the current trend of "back to the basics," and the infiltration of the concept of integers into the elementary school indicated a need for research into the placement of the integers in the elementary curricula. Since no prior research has been reported on the sequencing of the concept of integers, there was clearly a need for this particular study.

Procedures

Pilot Study

The study was designed to involve a treatment period of nine days; therefore the investigator constructed nine daily lessons and lesson plans, along

with two achievement tests, one written and one oral. This necessitated a pilot study, which was used to determine the appropriateness of: 1) the materials developed for teaching the concept of integers as a directed distance, 2) the grade level range for which the study and materials were designed, 3) the sequence of the materials, and 4) the achievement testing instruments. The study was also used as a means for determining the need for refinement of the materials and achievement testing instruments.

The pilot study used students from a different school than the formal study and consisted of six third grade, five fourth grade, and six fifth grade subjects each of whom were exposed to the sequence of learning activities on integers as a directed distance for 25 minutes each day for ten days. Prior to the learning activity sequence, the Laurendeau and Pinard instrument was used to determine each subject's level of left-right conceptualization.

Following the treatment period the written achievement test (group) was given, and each subject was also given the individual oral test, the responses to which were audio-taped. The tapes were used by the investigator to aid in evaluating the student's responses on the oral achievement test.

Using the results of the pilot study, it was determined that the grade level range was appropriate for the proposed study. According to scores on the achievement tests, the range appeared to include subjects able to understand the concept of integers as a directed distance and subjects unable to understand the concept. Using the subjects' reactions, it was also determined that the materials and their sequencing were appropriate for the concept and grade level with very little refinement being necessary.

The results of the preliminary study indicated a revision of the achievement testing instruments was needed. The written achievement instrument was consequently redesigned from short answer completion to multiple choice. The

wording of questions on the oral achievement test was clarified and the instrument was shortened.

Population

The subjects for the study were selected from students of an elementary school located in a suburb of Houston. The community from which the school obtains its student population of 1,100 consists of families from the low-middle to middle-middle socio-economic range. The subjects consisted of 49 third grade students, 56 fourth grade students and 52 fifth grade students.

Treatment and Data Collection

The nine day treatment was conducted by regular classroom teachers, so that a week prior to the beginning of the study, the investigator met with the teachers for two sessions lasting approximately one and a half hours each. During these sessions the following topics were discussed: 1) the inductive teaching method and how it was to be applied in the study, 2) the concept of integers, especially as a directed distance, 3) the terminology of the concept, 4) the sequence of the concept and materials, 5) the method of using and presenting the materials, and 6) the application of the lesson plans. The teachers were enthusiastic, cooperative, and appeared highly motivated in regard to being involved in the study.

Two days prior to the treatment period, each subject was tested to determine the level of left-right conceptualization. The test was administered by the investigator and a team of four doctoral students who had been trained in the use of the test.

The instructional period was organized around nine lessons using the inductive teaching strategy which were designed and developed by the investi-

gator. Each lesson plan included: 1) a generalization of the objective(s) of the lesson, 2) a list of materials to be used in the lesson, and 3) an abstract of the lesson. The lesson abstract included the sequencing of the content to be taught, suggested questions to be asked, activities to be included, and procedure for using the materials.

Following the instructional period, using a written and an oral achievement test designed by the investigator, two measures of achievement were collected on each subject.

Data Analysis

The data were analyzed using the following independent variables:

1. Grade Levels. The grades three through five were selected for the study as they included the age range from approximately eight years to eleven years. According to Piaget (1928), the conservation of number is achieved at approximately eight years and by age eleven formal operational thought is developed.

2. Age Levels. The age range of the subjects in the study was divided into six month intervals.

3. Levels of Left-Right Conceptualization. The subjects were classified according to their level of left-right conceptualization. This concept was selected due to the fact that a left-right directional system is often used in presenting the concept of integers as a directed distance.

4. Sex. Sex was used to classify the subjects since there appears to be a difference in quantitative ability between the sexes (Stone and Church, 1957).

The dependent variables in the study were the scores on the written and oral integer achievement tests.

The data were analyzed using two different experimental arrangements and their associated cell structures:

1. Using an experimental arrangement containing the independent variables grade, left-right conceptualization and sex, the vector of means for the integer achievement test scores were tested for significant difference over the levels of the independent variables and the interaction of the independent variables.

2. Using an experimental arrangement containing the independent variables age, left-right conceptualization and sex, the vector of means for the integer achievement test scores were tested for significant difference over the levels of age and the interaction of age with the other independent variables in the arrangement.

All data were analyzed at the .05 level of significance.

Findings and Discussion

Analysis of the data revealed that there is no significant difference in achievement on the concept of integers as a directed distance due to age, taken at six month intervals, and sex of the subjects. It may also be stated that the achievement was not affected by the interaction of: 1) left-right conceptualization and grade, 2) left-right conceptualization and sex, 3) left-right conceptualization and age, 4) grade and sex, 5) age and sex, and 6) left-right conceptualization, grade and sex.

Further analysis revealed that there was a significant difference in achievement on the concept of integers as a directed distance among levels of left-right conceptualization and the grades three through five. Using post hoc probes it was determined that the significant difference was caused by the written integer achievement test scores.

Using a Scheffe contrast, it was determined that there was a linear trend of the means over the levels of left-right conceptualization of the written integer achievement test scores. This linear trend of the means indicates that moving from one developmental level to the next in the three levels of left-right conceptualization would result in an increase in the performance on the written integer achievement test on the concept of integers as a directed distance.

It might be interesting to speculate why a significant difference in achievement on the concept of integers as a directed distance among the levels of left-right conceptualization was indicated by the written integer achievement test scores and not by the oral integer achievement test scores. The reason may be a result of the type of questions asked on the two achievement tests. The oral test contained questions which could be answered correctly without regard to the directions of left and right. The written test contained several questions in which the level of left-right conceptualization might very well affect the results of the response.

Using a Scheffe contrast, it was determined that there was both a linear and a quadratic trend of the means over the grade levels of the written integer achievement test scores. This phenomenon is possible due to the independence of the tests for the linear and quadratic trends. The linear trend implies that the performance on the written integer achievement test on the concept of integers as a directed distance increased from one grade level to the next in grades three through five. The quadratic trend implies that the performance on the written integer achievement test on the concept of integers as a directed distance increased from grade three to grade four but decreased from grade four to grade five.

Using the information from both the linear and quadratic trend, one can infer that an increase in performance on the written integer achievement test on the concept of integers as a directed distance is definitely indicated between grades three and four. The occurrence of an increase or a decrease in performance between grades four and five is questionable. However, by pooling the results of the linear and quadratic trends it would appear that the performance on the written integer achievement test peaks in grade four then tends to level off forming an asymtotic curve.

Table II

Means for Grade Level Pooled Over Left-Right Conceptualization and Sex

Level	n	ICW Means	ICO Means
1	49	19.90	7.35
2	56	27.79	10.73
3	52	25.60	11.37

The interpretations of the performance on the written integer achievement test among the grade levels are statistically correct based on the data collected; however, the interpretations should not be based on the data alone. The possible sources which contributed to the existing data structure should be examined. The fifth grade students having more acquired knowledge and maturity than the fourth grade students would be expected to perform better than the fourth grade students; however, an examination of the observed means

(Table II) on the written integer achievement test reveals that there was a decrease in performance between these grades. This decrease was verified statistically by the quadratic trend of the means indicated in the data analysis. One possible explanation for this occurrence might be the uncontrolled teacher variable. The investigator observed some of the instruction presented by the classroom teachers and due to the personality traits exhibited by the teacher was of the opinion that the instruction in the fifth grade was not as effective as that in the other two grades. This particular teacher spoke with a low keyed voice, seldom smiled, did not project enthusiasm and in general exhibited some of the tendencies of an introvert. It is the opinion of the investigator that this lack of enthusiasm in the presentation accounts in part for the decrease in performance indicated by the data between the fourth and fifth grades on the written integer achievement test.

A second potential source accounting for part of the decrease was the type of materials used in the study, i.e., the materials were primarily concrete and semi-concrete in nature. The students in the fifth grade whose intellectual development had reached the abstract stage may have considered the materials too immature and thus could have been "turned off" by them.

A third potential source accounting for the decrease may have been the procedures used in administering the written test to the three grades. The third and fourth grade teachers, being more extrovert in nature and from practice, may have been more expressive in reading the test items while administering the test to the students than the fifth grade teacher.

Using a first-order residual 'step-down' Scheffe contrast, it was determined that there was a quadratic trend of the means over the grade levels of the oral integer achievement test scores after statistically removing the effects of the written achievement test. The quadratic trend implies that the

performance on that portion of the oral integer achievement test on the concept of integers as a directed distance free from the effects of the written achievement test increased from grade three to grade four and decreased from grade four to grade five.

The quadratic trend of the residualized means of the oral integer achievement test, obtained by statistically removing the effects of the written integer achievement test, may be accounted for by the dominant quadratic trend occurring in the means on the written integer achievement test. The observed means on the oral integer achievement test appeared linear in nature (Table II); thus, it must have been the removal of the effects of the written integer achievement test which produced the quadratic trend. Therefore, the previously cited sources potentially accounting for the decrease in performance between the fourth and fifth grades on the written integer achievement test may also account for the indicated decrease in the oral integer achievement test after statistically removing the effects of the written integer achievement test.

Conclusions

Based on the analysis of the data in this study and the potential sources contributing to the data structure it would appear that teaching the concept of integers as a directed distance in the third grade would not be as advisable as in the fourth and fifth grades. Also, if the achievement on the concept is to be evaluated utilizing a written test, those students who have reached level 2 or 3 in the developmental levels of left-right conceptualization would have an advantage over those whose left-right developmental level was lower.

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